CS 133 : Automata Theory and Computability

Context-Free Grammars and Languages

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Day 12

Context-Free Grammars and Languages

Pushdown Automata

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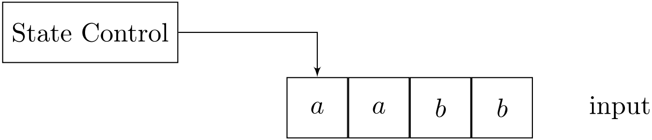
Schematic of a finite automaton:



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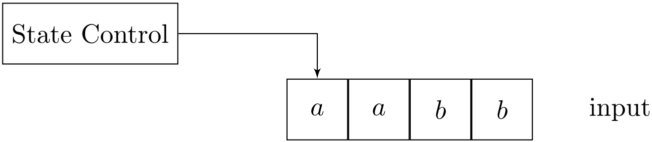
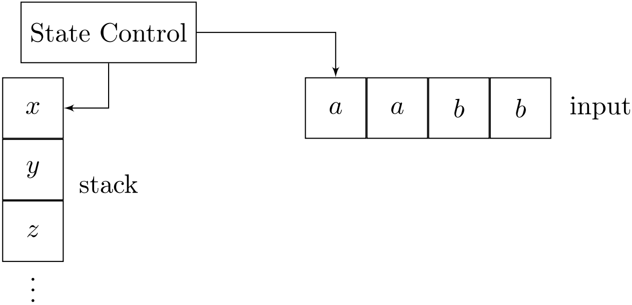


Schematic of a pushdown automaton:



Formal definition of PDA

Definition

A nondeterministic pushdown automaton (in short, PDA) is a 6-tuple $(Q, \Sigma, \Gamma, \delta, q_0, F)$, where Q, Σ, Γ and F are all finite sets, and

1. Q is the set of states,
2. Σ is the input alphabet,
3. Γ is the stack alphabet,
4. $\delta : Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \to \wp(Q \times \Gamma_\varepsilon)$ is the transition function,
5. $q_0 \in Q$ is the start state, and
6. $F \subseteq Q$ is the set of accept states.

Formal definition of PDA computation

A PDA M accepts input w for $w = w_1 w_2 \cdots w_m$, where each $w_i \in \Sigma_\varepsilon$ and sequences of states $r_0, r_1, \cdots, r_m \in Q$ and strings $s_0, s_1, \cdots, s_m \in \Gamma^*$ exist that satisfy the following conditions:

1. $r_0 = q_0$ and $s_0 = \varepsilon$,
2. For $i = 0, \cdots, m-1$, we have $(r_{i+1}, b) \in \delta(r_i, w_{i+1}, a)$, where $s_i = at$ and $s_{i+1} = bt$ for some $a, b \in \Gamma_\varepsilon$ and $t \in \Gamma^*$,
3. $r_m \in F$.

READ the previous conditions as:

1. M begins at start state with an empty stack.
2. M moves *properly* according to the state, stack, and next input symbol.
3. Accept state r_m occurs AFTER reading the final symbol w_m of w.

Example ($\{0^n 1^n \mid n \geq 0\}$)

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Let $M_1 = (Q, \Sigma, \Gamma, \delta, q_1, F)$ where

- $Q = \{q_1, q_2, q_3, q_4\}$,
- $\Sigma = \{0, 1\}$,
- $\Gamma = \{0, \$\}$,
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Input:	0			1			ε		
Stack:	0	\$	ε	0	\$	ε	0	\$	ε
q_1									$\{(q_2, \$)\}$
q_2			$\{(q_2, 0)\}$	$\{(q_3, \varepsilon)\}$					
q_3				$\{(q_3, \varepsilon)\}$				$\{(q_4, \varepsilon)\}$	
q_4									

Input:	0			1			ε		
Stack:	0	\$	ε	0	\$	ε	0	\$	ε
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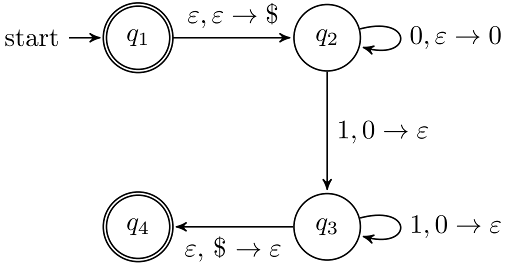
q_3				$\{(q_3, \varepsilon)\}$				$\{(q_4, \varepsilon)\}$	
q_4									

Input:	0			1			ε		
Stack:	0	\$	ε	0	\$	ε	0	\$	ε
q_1									$\{(q_2, \$)\}$
q_2			$\{(q_2, 0)\}$	$\{(q_3, \varepsilon)\}$					
q_3				$\{(q_3, \varepsilon)\}$				$\{(q_4, \varepsilon)\}$	
q_4									



Example

$\{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i = j \text{ or } i = k\}$

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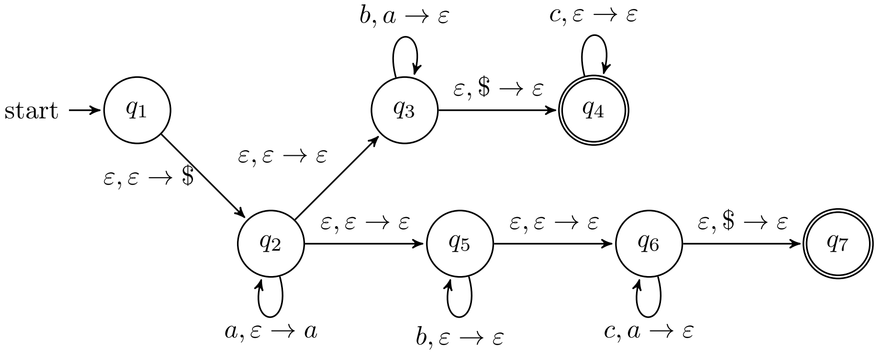
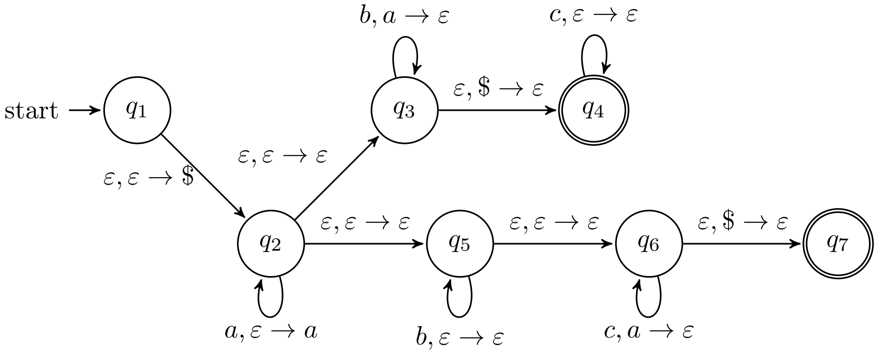


Example

$\{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i = j \text{ or } i = k\}$



Why is string *acc* not accepted despite arriving at q_7? Definition!

Example

$\{ww^R \mid w \in \{0,1\}^*\}$

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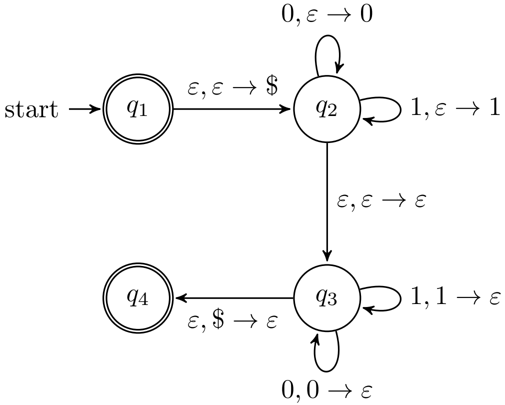

Try this ...

Construct PDAs that recognize the ff languages:

1. The set of palindromes over $\{0, 1\}$.
2. The set of balanced parentheses.
3. The language described by the regular expression $0^*1(0 \cup 1)^*$.

Fin (wakas)

Thanks for the attention.

Questions?

Reading assignment(s)

[Sipser 2005] Chapter 2.2

References:

[Sipser 2005] M. Sipser. *Introduction to the Theory of Computation*: 2ed. PWS Publishing Company, 2005.

[Hopcroft, Ullman 1979] J. Hopcroft, J. Ullman. *Introduction to Automata Theory, Languages, and Computation*: Addison-Wesley, 1979.

[Hopcroft et al 2001] J. Hopcroft, R. Motwani, J. Ullman. *Introduction to Automata Theory, Languages, and Computation*: Addison-Wesley, 2001.