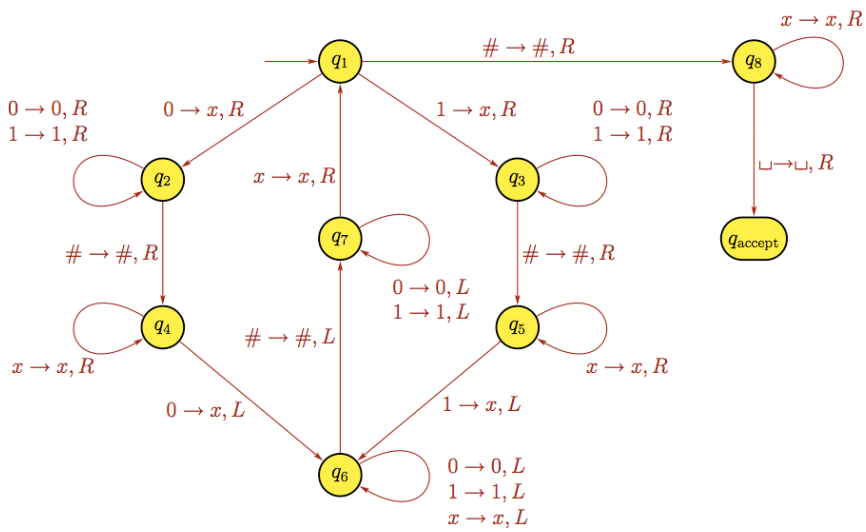
Do the following problems. Show your complete solutions. No solution, no point.

1. Given the following definition of a Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_1, q_{accept}, q_{reject})$:

   - $Q = \{q_1, q_2, \ldots, q_8, q_{accept}, q_{reject}\}$
   - $\Sigma = \{0, 1, \#\}$
   - $\Gamma = \Sigma \cup \{x, \sqcup\}$
   - δ :



Note: To simplify the figure, the reject state and transitions going to the reject state are not shown. Those transitions occur implicitly whenever a state lacks an outgoing transition for a particular symbol.

   Simulate the Turing machine on input (a) 010#1 and (b) 1#1.
   Use the concept of configurations to illustrate the transitions.

2. Let A be the set of all strings over the alphabet $\{a, b\}$ where the number of a's is twice as much as the number of b's. Construct the state diagram of a Turing machine that recognizes A.

3. Prove that the language $A\varepsilon_{CFG} = \{< G > \mid G \text{ is a CFG that generates } \varepsilon\}$ is decidable.

4. Use the fact that $ALL_{CFG} = \{< G > \mid G \text{ is a CFG and } L(G) = \Sigma^*\}$ is not a decidable language to prove that the following language is not decidable.

$$EQ_{CFG} = \{< G, H > \mid \text{G and H are CFGs and } L(G) = L(H)\}$$

5. Let L be a language that is not Turing-recognizable.
   Consider the language:

$$L' = \{w \mid w \text{ has odd length and } \ w \in L\} \cup \{w \mid w \text{ has even length and w } \notin L\}$$

   Show that L' and its complement $\overline{L'}$ cannot be both Turing-recognizable.