Do the following problems. Show your complete solutions. No solution, no point.

1. Use the pumping lemma for regular languages to prove that the language over the alphabet $\{a, b, c\}$,
$$B = \{a^m b^n c^n | m \geq 1, n \geq 0\}$$
is not regular.

2. Answer each part for the following context-free grammar G:
$$S \rightarrow Sc \mid Ab \mid Bc$$
$$A \rightarrow aAb \mid \varepsilon$$
$$B \rightarrow aB \mid \varepsilon$$
   (a) What are the variables of G?
   (b) What are the terminals of G?
   (c) What is the start variable of G?
   (d) Give three strings in $L(G)$.
   (e) Give **(i)** parse trees and **(ii)** leftmost derivations for each of the strings in **(d)**.
   (f) Give two strings **not** in $L(G)$.
   (g) Convert G into an equivalent context-free grammar in Chomsky Normal Form.
   (h) Describe $L(G)$.
   (i) Construct a pushdown-automaton that accepts $L(G)$.
   (j) Is the PDA you constructed deterministic? Explain why or why not.

3. Is the following grammar ambiguous? Why?
$$S \rightarrow AB \mid aaB$$
$$A \rightarrow a \mid Aa$$
$$B \rightarrow b$$
   Construct an unambiguous grammar which is equivalent to this grammar.

4. Consider the language
$$L = \{a^p\, b\, a^q\, b\, a^r \mid p, q, r \geq 0 \text{ and either } p = r \text{ or } q = r\}.$$
   (a) Give a context-free grammar for L. Be sure to specify G as a 4-tuple (V, Σ, R, S).
   (b) Give a PDA for L. You only need to draw the state diagram.