Do the following problems. Show your complete solutions. No solution, no point.

1. Prove that the language $A\varepsilon_{CFG} = \{< G > \mid G \text{ is a CFG that generates } \varepsilon\}$ is decidable.

2. Use the fact that $ALL_{CFG} = \{< G > \mid G \text{ is a CFG and } L(G) = \Sigma^*\}$ is not a decidable language to prove that the following language is not decidable.

$$EQ_{CFG} = \{< G, H > \mid \text{G and H are CFGs and } L(G) = L(H)\}$$

3. Let L be a language that is not Turing-recognizable.
Consider the language:

$$L' = \{w \mid w \text{ has odd length and } \ w \in L\} \cup \{w \mid w \text{ has even length and w } \ \notin L\}$$

Show that L' and its complement $\overline{L'}$ cannot be both Turing-recognizable.

4. Let $INDSET = \{< H, n > \mid H \text{ is an undirected graph containing an independent set of size } n\}$ where an independent set of size n is a set of n vertices with no edges between them.
Also, recall that $CLIQUE = \{< G, k > \mid G \text{ is an undirected graph with a } k\text{-clique}\}$ where a k-clique is a set of k vertices such that every pair of these vertices are connected by an edge.

    (a) Show that $INDSET$ is in NP.

    (b) Show that $INDSET$ is NP-complete by giving a polynomial-time reduction from $CLIQUE$.